Towards proof of new identity for Green functions in $N=1$ supersymmetric electrodynamics.

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Abstract

For the $N=1$ supersymmetric massless electrodynamics, regularized by higher derivatives, we describe a method, by which one can try to prove the new identity for the Green functions, which was proposed earlier. Using this method we show that some contribution to the new identity are really 0.

1 Introduction.

Investigation of quantum corrections in supersymmetric theories is an interesting and sometimes nontrivial problem. For example, in theories with the $N=1$ supersymmetry it is possible to suggest [1] a form of the β-function exactly to all orders of the perturbation theory. For the $N=1$ supersymmetric electrodynamics, which is considered in this paper, this β-function (that is called the exact Novikov, Shifman, Vainshtein and Zakharov (NSVZ) β-function) is

$$\beta(\alpha)=\frac{\alpha^2}{\pi}\Big(1-\gamma(\alpha)\Big), \tag{1}$$

where $\gamma(\alpha)$ is the anomalous dimension of the matter superfield. Obtaining such a β-function by methods of the perturbation theory appears unexpectedly complicated, although numerous verifications by explicit calculations up to the four-loop approximation [2] confirm this hypothesis. In these papers the β-function, defined as the derivative of divergence in the $\overline{MS}$-scheme, was calculated with the dimensional reduction. The result is that if the subtraction scheme is tuned by a special way, then it is possible to obtain the exact NSVZ β-function. Nevertheless, it is unclear, in what scheme such a β-function is obtained. The answer to this question in the three-loop approximation was given in Ref. [3]. According to this paper, the NSVZ β-function coincides with the Gell-Mann–Low function. We note that the most convenient method for the calculations is the higher covariant derivative regularization [4]. Unlike the dimensional regularization [5], it does

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not break supersymmetry, and, unlike the dimensional reduction [6], it is not inconsistent. Using the higher derivative regularization allows revealing an interesting feature of the quantum correction structure in supersymmetric theories, which was first noted in Ref. [3]: All integrals defining the Gell-Mann–Low function are integrals of total derivatives and can be easily calculated. A similar feature takes place in non-Abelian supersymmetric theories [7, 8]. (The calculations were made with a version of the higher derivative regularization, breaking the BRST-invariance and supplemented by a special subtraction scheme, which guarantees fulfilling the Slavnov–Taylor identities [9].) This feature was partially explained in Refs. [10, 11]. According to these papers, substituting solutions of the Slavnov–Taylor identities into the Schwinger–Dyson equations, it is possible to obtain the exact β-function if we suppose existence of a new identity for Green functions. The explicit calculations up to the four-loop approximation [12, 7] confirm this identity. A way of proving the new identity was proposed in Ref. [13]. However, it was based on the analysis of Feynman rules instead of strict functional methods. Moreover, it works only for a restricted class of diagrams. Nevertheless, an idea proposed in [13] can be strictly realized. This is demonstrated in this paper. We will see that it is possible to give a functional formulation for most equations, presented in Ref. [13]. The purpose of this paper is proposing of the method, which can be used for a strict proving (or disproving) the new identity in the massless $N = 1$ supersymmetric electrodynamics. As we will see later, using this method, it is possible to show that some contributions to the new identity are 0.

This paper is organized as follows.

In Sec. 2 we collect basic information about the $N = 1$ supersymmetric electrodynamics. In Sec. 3 we remind, how it is possible to calculate its β-function exactly to all orders, and also present a functional formulation of the new identity for Green functions, writing it as equality to 0 of some composite operators correlator. Calculation of this correlator is described in Sec. 4. The results are briefly discussed in the Conclusion. Some technical details of the calculations are presented in the Appendix.

2 $N = 1$ supersymmetric electrodynamics.

In this paper we consider the massless $N = 1$ supersymmetric electrodynamics, which is described in the superspace by the action

$$S = \frac{1}{4e^2}\mathrm{Re}\int d^4x\, d^2\theta\, W_a C^{ab} W_b + \frac{1}{4}\int d^4x\, d^4\theta\, \Big(\phi^* e^{2V}\phi + \tilde\phi^* e^{-2V}\tilde\phi\Big). \qquad (2)$$

Here ϕ and $\tilde\phi$ are chiral matter superfields and V is a real scalar superfield, which contains the gauge field A_μ as a component. The superfield W_a is a supersymmetric analogue of the gauge field stress tensor. In the Abelian case it is defined by

$$W_a = \frac{1}{4}\bar D^2 D_a V, \qquad (3)$$

where D_a and $\bar D_a$ are the right and left supersymmetric covariant derivatives respectively. (In this paper all left spinors we denote by a bar and $\bar D^2 \equiv \bar D^a \bar D_a$. Indexes are raised

and lowered by the charge conjugated matrix.) Action (2) is invariant under the gauge transformations

$$V \to V - \frac{1}{2}(\lambda + \lambda^*); \qquad \phi \to e^{\lambda}\phi; \qquad \tilde\phi \to e^{-\lambda}\tilde\phi, \tag{4}$$

where λ is an arbitrary chiral superfield.

In order to regularize model (2), it is possible to add the term with the higher derivatives

$$S_\Lambda = \frac{1}{4e^2}\mathrm{Re}\int d^4x\, d^2\theta\, W_a C^{ab}\frac{\partial^{2n}}{\Lambda^{2n}} W_b \tag{5}$$

to its action. It is important that in the Abelian case, the superfield W^a is gauge invariant; hence there are the usual (instead of covariant) derivatives in the regularizing term.

Model (2) can be standardly quantized. For this, it is convenient to use the supergraph technique (described in detail in book [14]) and to fix a gauge by adding the terms

$$S_{gf} = -\frac{1}{64e^2}\int d^4x\, d^4\theta\,\Big(V D^2\bar D^2\Big(1+\frac{\partial^{2n}}{\Lambda^{2n}}\Big)V + V\bar D^2 D^2\Big(1+\frac{\partial^{2n}}{\Lambda^{2n}}\Big)V\Big). \tag{6}$$

After such terms are added, a part of the action quadratic in the superfield V takes the simplest form

$$S_{gauge} + S_{gf} = \frac{1}{4e^2}\int d^4x\, d^4\theta\, V\partial^2\Big(1+\frac{\partial^{2n}}{\Lambda^{2n}}\Big)V. \tag{7}$$

In the Abelian case considered here, the diagrams containing ghost loops are absent. It is well known that adding the higher derivative term does not remove divergences in the one-loop diagrams. To regularize them, it is necessary to insert Pauli–Villars determinants in the generating functional [15].

The renormalized action for the considered model is

$$S_{ren} = \frac{1}{4e^2}Z_3(e,\Lambda/\mu)\,\mathrm{Re}\int d^4x\, d^2\theta\, W_a C^{ab}\Big(1+\frac{\partial^{2n}}{\Lambda^{2n}}\Big)W_b + \\ + Z(e,\Lambda/\mu)\,\frac{1}{4}\int d^4x\, d^4\theta\,\Big(\phi^* e^{2V}\phi + \tilde\phi^* e^{-2V}\tilde\phi\Big). \tag{8}$$

Hence, the generating functional can be written as

$$Z = \int DV\, D\phi\, D\tilde\phi\,\prod_i \Big(\det PV(V,M_i)\Big)^{c_i}\exp\Big(i(S_{ren} + S_{gf} + S_S + S_{\phi_0})\Big), \tag{9}$$

where the renormalized action S_{ren} is given by Eq. (8) and the action for gauge fixing terms is given by Eq. (6). (It is convenient to replace e with the bare coupling constant e_0 in it that we will suppose below.) The Pauli–Villars determinants are defined by

$$\Big(\det PV(V,M)\Big)^{-1} = \int D\Phi\, D\tilde\Phi\,\exp\Big(iS_{PV}\Big), \tag{10}$$

where

$$S_{PV} \equiv Z(e,\Lambda/\mu)\frac{1}{4}\int d^4x\, d^4\theta\, \Big(\Phi^* e^{2V}\Phi +
+\tilde\Phi^* e^{-2V}\tilde\Phi\Big) + \frac{1}{2}\int d^4x\, d^2\theta\, M\tilde\Phi\Phi + \frac{1}{2}\int d^4x\, d^2\bar\theta\, M\tilde\Phi^*\Phi^*, \qquad (11)$$

and the coefficients c_i satisfy the conditions

$$\sum_i c_i = 1; \qquad \sum_i c_i M_i^2 = 0. \qquad (12)$$

Below, we assume that $M_i = a_i\Lambda$, where a_i are some constants. Inserting the Pauli-Villars determinants allows cancelling the remaining divergences in all one-loop diagrams, including diagrams containing counterterm insertions.

The terms with sources are written as

$$S_S = \int d^4x\, d^4\theta\, JV + \int d^4x\, d^2\theta\,\Big(j\,\phi + \tilde j\,\tilde\phi\Big) + \int d^4x\, d^2\bar\theta\,\Big(j^*\phi^* + \tilde j^*\tilde\phi^*\Big). \qquad (13)$$

Moreover, in generating functional (9), we introduce the expression

$$S_{\phi_0} = \frac{1}{4}\int d^4x\, d^4\theta\,\Big(\phi_0^*\, e^{2V}\phi + \phi^*\, e^{2V}\phi_0 + \tilde\phi_0^*\, e^{-2V}\tilde\phi + \tilde\phi^*\, e^{-2V}\tilde\phi_0\Big), \qquad (14)$$

where ϕ_0, ϕ_0^*, $\tilde\phi_0$, and $\tilde\phi_0^*$ are scalar superfields that are not chiral or antichiral. Generally, it is not necessary to introduce the term S_{ϕ_0} in the generating functional, but the presence of the parameters ϕ_0 e.t.c. will be needed later.

In our notation, the generating functional for the connected Green functions is written as

$$W = -i\ln Z, \qquad (15)$$

and the effective action is obtained via the Legendre transformation

$$\Gamma = W - \int d^4x\, d^4\theta\, J\mathbf{V} - \int d^4x\, d^2\theta\,\Big(j\,\phi + \tilde j\,\tilde\phi\Big) - \int d^4x\, d^2\bar\theta\,\Big(j^*\phi^* + \tilde j^*\tilde\phi^*\Big), \qquad (16)$$

where the sources J, j, and $\tilde j$ should be expressed in terms of fields $\mathbf{V}$, ϕ, and $\tilde\phi$ using the equations

$$\mathbf{V} = \frac{\delta W}{\delta J}; \qquad \phi = \frac{\delta W}{\delta j}; \qquad \tilde\phi = \frac{\delta W}{\delta\tilde j}. \qquad (17)$$

(The argument of the effective action, corresponding to the field V, we denoted by the bold letter for convenience of the notation.)

In this paper we will mostly calculate the Gell-Mann–Low function, which is determined by dependence of the two-point Green function on the momentum. In order to construct it, we write terms in the effective action, corresponding to the renormalized two-point Green function, in the form

$$\Gamma^{(2)}_V = -\frac{1}{16\pi}\int \frac{d^4p}{(2\pi)^4}\, d^4\theta\, \mathbf{V}(-p)\, \partial^2\Pi_{1/2}\mathbf{V}(p)\, d^{-1}(\alpha,\mu/p), \tag{18}$$

where $\Pi_{1/2}$ is a supersymmetric projection operator, and α is a renormalized coupling constant. The Gell-Mann–Low function, denoted by $\beta(\alpha)$, is defined by

$$\beta\Big(d(\alpha,\mu/p)\Big) = \frac{\partial}{\partial\ln p} d(\alpha,\mu/p). \tag{19}$$

It is well known that the Gell-Mann–Low function is scheme independent.

The anomalous dimension is defined similarly. First we consider the two-point Green function for the matter superfield

$$\Gamma^{(2)}_\phi = \frac{1}{4}\int \frac{d^4p}{(2\pi)^4}\, d^4\theta\, \Big(\phi^*(-p,\theta)\,\phi(p,\theta) + \tilde\phi^*(-p,\theta)\,\tilde\phi(p,\theta)\Big)\, G_{ren}(\alpha,\mu/p), \tag{20}$$

where

$$G_{ren}(\alpha,\mu/p) = Z(\alpha_0,\Lambda/\mu) G(\alpha_0,\Lambda/p), \tag{21}$$

and Z denotes the renormalization constant for the matter superfield. Then the anomalous dimensions is defined by

$$\gamma\Big(d(\alpha,\mu/p)\Big) = -\frac{\partial}{\partial\ln p}\ln G_{ren}(\alpha,\mu/p). \tag{22}$$

3 Calculation of the matter superfields contribution.

To calculate a contribution of matter superfields it is convenient to use an approach, based on substituting solutions of Slavnov–Taylor identities into the Schwinger–Dyson equations. The Schwinger–Dyson equations in the considered theory can be written as

$$\frac{\delta\Gamma}{\delta\mathbf{V}_x} = \frac{1}{2}\Big\langle \phi^*_x e^{2V_x}\phi_x - \tilde\phi^*_x e^{2V_x}\tilde\phi_x + \Big(\phi^*_{0x} e^{2V_x}\phi_x - \tilde\phi^*_{0x} e^{2V_x}\tilde\phi_x + \text{h.c.}\Big) - (PV)\Big\rangle; \tag{23}$$

$$\frac{\delta\Gamma}{\delta\phi^*_x} = -\frac{D^2_x}{2}\Big\langle e^{2V_x}\phi_x + e^{2V_x}\phi_{0x}\Big\rangle = -2D^2_x\frac{\delta\Gamma}{\delta\phi^*_{0x}} - \frac{D^2_x}{2}\Big\langle e^{2V_x}\phi_{0x}\Big\rangle, \tag{24}$$

where (PV) in the first equation denotes contributions of the Pauli–Villars fields. Due to the second equation, derivatives with respect to the additional sources ϕ_0 and $\tilde\phi_0$ are very similar to derivatives with respect to the fields ϕ and $\tilde\phi$. The difference of Feynman diagrams is that there is the operator $\bar D^2$ on external ϕ-lines, and there is no this operator on external ϕ_0-lines.

Let us differentiate the first Schwinger–Dyson equation with respect to $\mathbf{V}_y$ and set all fields and sources to 0:

$$\frac{\delta^2\Gamma}{\delta\mathbf{V}_y\delta\mathbf{V}_x} = \frac{2}{i}\frac{\delta}{\delta\mathbf{V}_y}\Big(\frac{\delta}{\delta j^*_x}\frac{\delta W}{\delta\phi^*_{0x}} - \frac{\delta}{\delta\tilde j^*_x}\frac{\delta W}{\delta\tilde\phi^*_{0x}}\Big). \tag{25}$$

Then we commute the variational derivative with respect to the field $\mathbf{V}_y$ and derivatives with respect to the sources j^* and $\tilde{j}^*$. Moreover, we take into account that the fields ϕ_0^* are some parameters. Hence,

$$\frac{\delta W}{\delta\phi_0^*} = \frac{\delta\Gamma}{\delta\phi_0^*}. \tag{26}$$

As a result we obtain

$$\frac{\delta^2\Gamma}{\delta\mathbf{V}_y\delta\mathbf{V}_x} = \frac{2}{i}\Big(\Big[\frac{\delta}{\delta\mathbf{V}_y},\frac{\delta}{\delta j_x^*}\Big]\frac{\delta\Gamma}{\delta\phi_{0x}^*} - \Big[\frac{\delta}{\delta\mathbf{V}_y},\frac{\delta}{\delta\tilde{j}_x^*}\Big]\frac{\delta\Gamma}{\delta\tilde{\phi}_{0x}^*} + \frac{\delta}{\delta j_x^*}\frac{\delta^2\Gamma}{\delta\phi_{0x}^*\delta\mathbf{V}_y} - \frac{\delta}{\delta\tilde{j}_x^*}\frac{\delta^2\Gamma}{\delta\tilde{\phi}_{0x}^*\delta\mathbf{V}_y}\Big). \tag{27}$$

Taking into account that the source j^* is an antichiral field, a contribution of the last two terms to the effective action can be written as

$$\begin{aligned}
&-i\int d^8x\,d^8y\,\mathbf{V}_y\mathbf{V}_x\frac{\delta}{\delta j_x^*}\frac{\delta^2\Gamma}{\delta\phi_{0x}^*\delta\mathbf{V}_y} = i\int d^8x\,d^8y\,\mathbf{V}_y\mathbf{V}_x\frac{D_x^2\bar{D}_x^2}{16\partial^2}\frac{\delta}{\delta j_x^*}\frac{\delta^2\Gamma}{\delta\phi_{0z}^*\delta\mathbf{V}_y}\Big|_{z=x} = \\
&= i\int d^8x\,d^8y\,\mathbf{V}_y\Big(\mathbf{V}_x\frac{\bar{D}_x^2}{16\partial^2}\frac{\delta}{\delta j_x^*}D_x^2\frac{\delta^2\Gamma}{\delta\phi_{0x}^*\delta\mathbf{V}_y} + D_x^2\mathbf{V}_x\frac{\bar{D}_x^2}{16\partial^2}\frac{\delta}{\delta j_x^*}\frac{\delta^2\Gamma}{\delta\phi_{0x}^*\delta\mathbf{V}_y} + \\
&+D_x^b\mathbf{V}_x\frac{\bar{D}_x^2}{8\partial^2}\frac{\delta}{\delta j_x^*}D_{bx}\frac{\delta^2\Gamma}{\delta\phi_{0x}^*\delta\mathbf{V}_y}\Big).
\end{aligned} \tag{28}$$

The first term in this expression and a similar term with a commutator in Eq. (27) can be expressed in terms of the usual Green functions (which do not contain the additional sources) using the Schwinger–Dyson equations for matter superfields. They was calculated, for example, in Ref. [11]. The result is (taking into account a similar contribution of the fields $\tilde{\phi}$)

$$\begin{aligned}
&\frac{d}{d\ln\Lambda}\frac{\delta^2\Gamma}{\delta\mathbf{V}_y\delta\mathbf{V}_x}\Big|_{p=0} = \partial^2\Pi_{1/2}\delta^8_{xy}\int\frac{d^4q}{(2\pi)^4}\frac{d}{d\ln\Lambda}\Big\{\frac{1}{q^2}\frac{d}{dq^2}\ln(q^2G^2) - (PV)\Big\} + \\
&+\text{other contributions},
\end{aligned} \tag{29}$$

According to Refs. [10, 11], if all other contributions to the considered Green function are 0, then the presented expression corresponds to the exact NSVZ β-function.

However, the last two terms in Eq. (28) can not be already written in terms of the usual Green functions using the Schwinger–Dyson equations for matter superfields. Moreover, it is possible to find out [10, 11] that they are expressed in terms of functions that can not be found from the Slavnov–Taylor identities. However, explicit calculations show that they are always 0. Thus, we should propose the existence of a new identity

$$\begin{aligned}
&\frac{d}{d\ln\Lambda}\int d^8x\,d^8y\,\mathbf{V}_y\Big(D_x^2\mathbf{V}_x\frac{\bar{D}_x^2}{2\partial^2}\frac{\delta}{\delta j_x^*}\frac{\delta^2\Gamma}{\delta\phi_{0x}^*\delta\mathbf{V}_y} + D_x^b\mathbf{V}_x\frac{\bar{D}_x^2}{\partial^2}\frac{\delta}{\delta j_x^*}D_{bx}\frac{\delta^2\Gamma}{\delta\phi_{0x}^*\delta\mathbf{V}_y}\Big) + \\
&+\text{similar terms with } \tilde{j} \text{ and } \tilde{\phi}_0 = 0.
\end{aligned} \tag{30}$$

In order to rewrite these terms in a more visual form, we again use the Schwinger–Dyson equation for the gauge field. The result is

$$\frac{d}{d\ln\Lambda}\int d^8x\,d^8y\,\mathbf{V}_y\Big\langle\Big(D_x^2\mathbf{V}_x\frac{\bar D_x^2}{2\partial^2}\phi_x^* e^{2V_x}\phi_x + D_x^b\mathbf{V}_x\frac{D_{xb}\bar D_x^2}{\partial^2}\phi_x^* e^{2V_x}\phi_x\Big)\phi_y^* e^{2V_y}\phi_y\Big\rangle +$$
$$+\text{similar terms with } \tilde\phi = 0. \qquad (31)$$

This equation is a functional form of the new identity for Green functions. We remark that products of two-point correlators, which appear deriving this identity, are 0. For example,

$$\frac{1}{4}\Big\langle D^b\mathbf{V}_x\frac{D_b\bar D^2}{\partial^2}\phi_x^* e^{2V_x}\phi_x + D^2\mathbf{V}_x\frac{\bar D^2}{2\partial^2}\phi_x^* e^{2V_x}\phi_x\Big\rangle =$$
$$= -iD^b\mathbf{V}_x\frac{D_{xb}\bar D_x^2}{\partial^2}\frac{\delta}{\delta j_x^*}\frac{\delta\Gamma}{\delta\phi_{0z}^*}\Big|_{z=x} - iD^2\mathbf{V}_x\frac{\bar D_x^2}{2\partial^2}\frac{\delta}{\delta j_x^*}\frac{\delta\Gamma}{\delta\phi_{0z}^*}\Big|_{z=x} = 0. \qquad (32)$$

(To verify the last equality, it is possible to express the derivative with respect to the source in terms of derivatives with respect to fields and to use explicit expression for the two-point Green functions. Equality to 0 is obtained, because the expression $\hat P_x\delta^8_{xy}|_{x=y}$ is not 0 only if the operator $\hat P$ contains 4 spinor derivatives.) Due to the similar reasons, the terms obtained, if we differentiate the fields ϕ_0 in the Schwinger–Dyson equation for the gauge field, and the terms containing both ϕ and $\tilde\phi$ are 0.

Then, we use the identity

$$\phi^*\mathbf{V} = -\frac{D^2\bar D^2}{16\partial^2}\phi^*\mathbf{V} = -\frac{D^2\bar D^2}{16\partial^2}(\phi^*\mathbf{V}) + \frac{1}{16\partial^2}\phi^* D^2\bar D^2\mathbf{V} + \frac{\bar D^a}{8\partial^2}\phi^* D^2\bar D_a\mathbf{V} +$$
$$+\frac{\bar D^2}{16\partial^2}\phi^* D^2\mathbf{V} - i(\gamma^\mu)_{ab}\frac{\partial_\mu}{2\partial^2}\phi^* D^a\bar D^b\mathbf{V} + \frac{D^a\bar D^2}{8\partial^2}\phi^* D_a\mathbf{V}. \qquad (33)$$

In our notation $(\gamma^\mu)_{ab} \equiv -(\gamma^\mu)_a{}^c C_{cb}$, where $(\gamma^\mu)_a{}^c$ are usual γ-matrices, and C is a charge conjugation matrix.

In Appendix A we show that the first term in this expression does not contribute to the new identity. Some other terms are also 0. To verify this, we note that any third power of the chiral derivative can be written as an expression containing the derivative ∂_μ, which does not act on the background field $\mathbf{V}$, because we consider the limit of zero external momentum. Therefore, the new identity can be rewritten as

$$\frac{d}{d\ln\Lambda}\int d^8x\,d^8y\,\Big(D^a\mathbf{V}_x i(\gamma^\mu)_{bc}D^b\bar D^c\mathbf{V}_y\Big\langle\frac{D_a\bar D^2}{\partial^2}\phi_x^* e^{2V_x}\phi_x\frac{\partial_\mu}{2\partial^2}\phi_y^* e^{2V_y}\phi_y\Big\rangle +$$
$$+D^a\mathbf{V}_x D^b\mathbf{V}_y\Big\langle\frac{D_a\bar D^2}{\partial^2}\phi_x^* e^{2V_x}\phi_x\frac{D_b\bar D^2}{8\partial^2}\phi_y^* e^{2V_y}\phi_y\Big\rangle\Big) = 0. \qquad (34)$$

The next section of the paper is devoted to a possible way for proving this identity.

4 Way of proving the new identity

Now, let us describe how one can try to prove the new identity for Green functions. Actually the idea was formulated in Ref. [13], analyzing the Feynman rules. Here we will use strict functional methods. Moreover, there is a mistake in a sign in Ref. [13]. To correct this error, it is necessary to slightly modify the proof.

It is convenient to write the new identity in form (34). The functional integral over the matter fields is Gaussian and can be calculated explicitly. We will need the equation

$$\int dx_1\ldots dx_n\, x_a x_b x_c x_d \exp\Big(-\frac{1}{2}x_i A_{ij} x_j\Big) = \\ = \mbox{const}\,(\det A)^{-1/2}\Big(A^{-1}_{ab}A^{-1}_{cd} + A^{-1}_{ac}A^{-1}_{bd} + A^{-1}_{ad}A^{-1}_{bc}\Big). \qquad (35)$$

In the massless case A is the operator

$$D^2 e^{2V}\bar D^2 = D^2\bar D^2 + D^2(e^{2V}-1)\bar D^2. \qquad (36)$$

The operator inverse to A by definition satisfies the condition

$$\frac{1}{D^2 e^{2V}\bar D^2} D^2 e^{2V}\bar D^2 = D^2 e^{2V}\bar D^2 \frac{1}{D^2 e^{2V}\bar D^2} = -\frac{D^2\bar D^2}{16\partial^2} \equiv \hat 1. \qquad (37)$$

It can be easily constructed explicitly:

$$\frac{1}{D^2 e^{2V}\bar D^2} = \Big(1 - \frac{1}{16\partial^2}D^2(e^{2V}-1)\bar D^2\Big)^{-1}\frac{D^2\bar D^2}{16^2\partial^4} = \\ = \sum_{n=0}^{\infty}\Big(\frac{1}{16\partial^2}D^2(e^{2V}-1)\bar D^2\Big)^n \frac{D^2\bar D^2}{16^2\partial^4}. \qquad (38)$$

After the functional integration over matter superfields by using Eq. (35), the left hand side of the new identity (up to an insignificant factor) becomes

$$\mbox{Tr}\,\frac{d}{d\ln\Lambda}\Big\langle \frac{1}{D^2 e^{2V}\bar D^2} D^a (D_a\mathbf{V}) e^{2V}\bar D^2\Big(i(\gamma_\mu)_{bc}\frac{1}{D^2 e^{2V}\bar D^2}\frac{\partial_\mu}{\partial^2}D^2(\bar D^b D^c\mathbf{V})e^{2V}\bar D^2 - \\ -\frac{4}{D^2 e^{2V}\bar D^2} D^b (D_b\mathbf{V}) e^{2V}\bar D^2\Big)\Big\rangle. \qquad (39)$$

The trace includes the integration over the superspace:

$$\mbox{Tr}\hat A \equiv \int d^8x\, A_{xx}, \qquad (40)$$

and the angular brackets here and below denote taking the vacuum expectation value by the functional integration only with respect to the gauge field.

We remark that it is also possible to use a brief notation introduced in Ref. [13]:

$$* \equiv -\frac{4}{D^2 e^{2V}\bar D^2};\qquad (\bar I_1)^a \equiv \frac{1}{2}D^2 e^{2V}\bar D^a;\qquad (I_1)^a \equiv \frac{1}{2}D^a e^{2V}\bar D^2;\\ (I_2) \equiv \frac{1}{4}e^{2V}\bar D^2;\qquad (\bar I_2) \equiv \frac{1}{4}D^2 e^{2V};\qquad (I_2)^{ab} \equiv D^a e^{2V}\bar D^b;\\ (I_3)^a \equiv \frac{1}{2}D^a e^{2V};\qquad (\bar I_3)^a \equiv \frac{1}{2}e^{2V}\bar D^a;\qquad (I_0) \equiv \frac{1}{4}D^2 e^{2V}\bar D^2. \qquad (41)$$

In Ref. [13] these expressions were defined differently, since in that paper we did not use functional methods. Nevertheless, main formulas, strictly derived in this paper, are similar to formulas in [13]. Using this notation, Eq. (39) can be graphically presented as a sum of two effective diagrams, presented in Fig. 1.

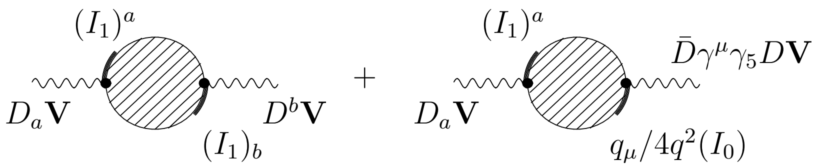

Figure 1: Graphical form of Eq. (39).

If the new identity is true, than expression (39) is 0. In order to verify this, we have to commute factors containing the background field with the expression $(D^2 e^{2V}\bar{D}^2)^{-1}$. This can be made by using the identities

$$\hat{1}\cdot[A^{-1},X] = A^{-1}[X,A]A^{-1} + A^{-1}[\hat{1},X];$$
$$[A^{-1},X]\cdot\hat{1} = A^{-1}[X,A]A^{-1} + [\hat{1},X]A^{-1}. \tag{42}$$

If $X_{yz} = X_y\delta^8_{yz}$ and $A^{-1}_{xy} = \hat{O}_x\delta^8_{xy}$, where $\hat{O}$ is an operator, then

$$[A^{-1},X]_{xz} = A^{-1}_{xz}X_z - X_x A^{-1}_{xz} = \hat{O}_x\delta^8_{xz}X_z - X_x\hat{O}_x\delta^8_{xz} = \hat{O}_x(X_x\delta^8_{xz}) - X_x\hat{O}_x\delta^8_{xz}. \tag{43}$$

To calculate commutators with other expressions containing the supersymmetric covariant derivatives, below we will use the following identities:

$$\bar{D}^2 X = X\bar{D}^2 - 2(-1)^{P_x}(\bar{D}_a X)\bar{D}^a + (\bar{D}^2 X);$$
$$XD^2 = D^2 X - 2D^a(D_a X) + (D^2 X);$$
$$\bar{D}^a X = (-1)^{P_X} X\bar{D}^a + (\bar{D}^a X);$$
$$XD^a = (-1)^{P_X} D^a X - (-1)^{P_X}(D^a X), \tag{44}$$

where P_X denotes the Grassmanian parity of the expression X, which is 0(mod 2) or 1(mod 2).

Now let us proceed to calculating expression (39). For this purpose we note that acting on $\mathbf{V}$ the derivatives ∂_μ gives 0, because the momentum of this field is 0. Due to the same reason, more than 4 spinor derivatives can not act on the field $\mathbf{V}$. Moreover, the new identity contributes only to the transversal part of the gauge field Green function. (This follows from the Ward identity and the result of calculating two-point Green function using the Schwinger–Dyson equations.) Hence, we may keep only terms with 2 derivatives D and 2 derivatives $\bar{D}$ acting on the background field.

Therefore, to simplify the calculations, it is possible to substitute formally the background field $\mathbf{V}$ for $\bar{\theta}^2\theta^2$. In this case the calculations are automatically made in the limit of zero external momentum, because this expression is independent of the coordinates. Since

$$\int d^4\theta\, (\bar\theta^2\theta^2)\partial^2\Pi_{1/2}(\bar\theta^2\theta^2) = -\frac{1}{2}\int d^4\theta\,\bar\theta^2\theta^2 = -2, \qquad (45)$$

after this substitution

$$\int \frac{d^4p}{(2\pi)^4}\, d^4\theta\, \mathbf{V}\partial^2\Pi_{1/2}\mathbf{V}\int \frac{d^4q}{(2\pi)^4}\frac{f}{q^2G} \to -2\int \frac{d^4p}{(2\pi)^4}\,\frac{d^4q}{(2\pi)^4}\,\frac{f}{q^2G}. \qquad (46)$$

Thus, having obtained formally the right hand side, it is possible to uniquely restore the left hand side. These speculations can be easily verified by calculating the simplest Feynman diagrams, for example, one- or two-loop diagrams.

Using the described method, it is easy to prove that the first term in Eq. (39) is 0. Really, after the substitution $\mathbf{V}\to\theta^2\bar\theta^2$ it is written as

$$T_1 \equiv i(\gamma_\mu)_{bc}\mathrm{Tr}\,\frac{d}{d\ln\Lambda}\Big\langle \frac{1}{D^2e^{2V}\bar D^2}D^a\theta_a\bar\theta^2e^{2V}\bar D^2\frac{1}{D^2e^{2V}\bar D^2}\frac{\partial_\mu}{\partial^2}D^2\bar\theta^b\theta^c e^{2V}\bar D^2\Big\rangle \qquad (47)$$

up to an insignificant constant. The derivative with respect to $\ln\Lambda$ in this equation acts only on propagators of the gauge superfields, which are obtained calculating the functional integrals over the fields V. Due to the presence of this derivative, all considered expression are well defined. Really, in the momentum representation we first make the differentiation with respect to $\ln\Lambda$, and then perform integration with respect to loop momentums. Since trace and differentiation with respect to $\ln\Lambda$ commute, the operator $\mathrm{Tr}\,d/d\ln\Lambda$ is invariant under cyclic replacements.

We note that the right spinor θ in Eq. (47) can be moved in any place. Really, calculating the trace we obtain

$$\int d^4x\, d^4\theta\, \hat P\delta^8_{12}\Big|_{\theta_1=\theta_2=\theta;\, x_1=x_2=x}, \qquad (48)$$

where the operator $\hat P$ is a product of covariant derivatives and also two factors $\theta_a\bar\theta^2$ and $\bar\theta^b\theta^c$, which are contained in the considered trace. Commuting θ and $\bar\theta$ with covariant derivatives, it is possible to shift all θ and $\bar\theta$ so that they will be on the left of the covariant derivatives. It is well known that the action of the spinor covariant derivatives on $\delta^4(\theta_1-\theta_2)$ in coinciding points is an expression independent of θ. (In order to verify this, it is necessary to make a number of the spinor derivatives less than or equal to 4 using commutation relations. If a number of the derivatives is less than 4, then acting on the δ-function in coinciding points gives 0, and if this number is 4, this gives an expression independent of θ.) Therefore, a nontrivial contribution to Eq. (48) can be obtained only if there is the expression

$$\int d^4\theta\,\theta^2\bar\theta^2 = 4 \qquad (49)$$

on the left of the covariant derivatives. But the anticommutator of the covariant derivative with θ is a constant. Therefore, all terms with commutators are 0, because a degree of θ in them is less than 2. This means that the right spinor θ in the considered expression can be moved in any place. In particular, both θ can be put together. Then, using the identity $\theta^c\theta_a=\delta_a^c\,\theta^2/2$ and taking into account that D and $\bar\theta^b$ anticommute, the considered expression can be rewritten as

$$T_1 = \frac{i}{2}(\gamma_\mu)_{ab}\mathrm{Tr}\,\frac{d}{d\ln\Lambda}\Big\langle\frac{1}{D^2e^{2V}\bar D^2}D^a\theta^2\bar\theta^2e^{2V}\bar D^2\frac{1}{D^2e^{2V}\bar D^2}\frac{\partial_\mu}{\partial^2}\bar\theta^b D^2e^{2V}\bar D^2\Big\rangle =$$
$$= -\frac{i}{2}(\gamma_\mu)_{ab}\mathrm{Tr}\,\frac{d}{d\ln\Lambda}\Big\langle\frac{D^2\bar D^2}{16\partial^2}D^a\theta^2\bar\theta^2e^{2V}\bar D^2\frac{1}{D^2e^{2V}\bar D^2}\frac{\partial_\mu}{\partial^2}\bar\theta^b\Big\rangle. \qquad (50)$$

Moving $\bar\theta^b$ using commutation relations (44) to the right until it will be multiplied by $\bar\theta^2$, we find

$$T_1 = -i(\gamma_\mu)_{ab}\mathrm{Tr}\,\frac{d}{d\ln\Lambda}\Big\langle\frac{D^2\bar D^b}{16\partial^2}D^a\theta^2\bar\theta^2e^{2V}\bar D^2\frac{1}{D^2e^{2V}\bar D^2}\frac{\partial_\mu}{\partial^2}\Big\rangle =$$
$$= -\mathrm{Tr}\,\frac{d}{d\ln\Lambda}\Big\langle\frac{\partial^\mu}{8\partial^2}D^2\theta^2\bar\theta^2e^{2V}\bar D^2\frac{1}{D^2e^{2V}\bar D^2}\frac{\partial_\mu}{\partial^2}\Big\rangle. \qquad (51)$$

We note that $\theta^2\bar\theta^2$ can be shifted to an arbitrary place of this expression, since a commutation of θ or $\bar\theta$ with the supersymmetric covariant derivatives decreases a degree of θ or $\bar\theta$ on 1, and the integral over $d^4\theta$ is not 0 only if it acts on the fourth degree. Hence, the first term in Eq. (39) can be rewritten as

$$T_1 = -\mathrm{Tr}\,\frac{d}{d\ln\Lambda}\Big\langle\theta^2\bar\theta^2\frac{\partial^\mu}{8\partial^2}D^2e^{2V}\bar D^2\frac{1}{D^2e^{2V}\bar D^2}\frac{\partial_\mu}{\partial^2}\Big\rangle =$$
$$= \mathrm{Tr}\,\frac{d}{d\ln\Lambda}\Big\langle\theta^2\bar\theta^2\frac{\partial^\mu}{16\partial^2}\frac{D^2\bar D^2}{8\partial^2}\frac{\partial_\mu}{\partial^2}\Big\rangle = 0, \qquad (52)$$

where we take into account that the last expression is independent of Λ and disappears after differentiating with respect to $\ln\Lambda$. Really, in the momentum representation this expression is proportional to

$$\int\frac{d^4q}{(2\pi)^4}\frac{d}{d\ln\Lambda}\frac{1}{q^4} = 0. \qquad (53)$$

Presence of the derivative with respect to $\ln\Lambda$ guarantees that this expression is well defined. (We take the trace after the differentiation.)

Similarly we can try to prove that the second term in Eq. (39) is 0. However, the corresponding calculation is much more complicated. First, as earlier we make the formal substitution $\mathbf{V}\to\theta^2\bar\theta^2$, after which the second term in Eq. (39) will be proportional to

$$T_2 \equiv \mathrm{Tr}\,\frac{d}{d\ln\Lambda}\Big\langle\frac{1}{D^2e^{2V}\bar D^2}D^a\theta_a\bar\theta^2e^{2V}\bar D^2\frac{1}{D^2e^{2V}\bar D^2}D^b\theta_b\bar\theta^2e^{2V}\bar D^2\Big\rangle. \qquad (54)$$

Similar to calculating the first term, we can put together the right spinors θ:

$$T_2 = -\frac{1}{2}\mathrm{Tr}\,\frac{d}{d\ln\Lambda}\Big\langle\frac{1}{D^2e^{2V}\bar D^2}D^a\theta^2\bar\theta^2e^{2V}\bar D^2\frac{1}{D^2e^{2V}\bar D^2}D_a\bar\theta^2e^{2V}\bar D^2\Big\rangle. \qquad (55)$$

We also put together the factors $\bar\theta$, commuting them with the covariant derivatives. After some simple transformations the result can be written as

$$
\begin{aligned}
T_2 = \mathrm{Tr}\,\frac{d}{d\ln\Lambda}\,\theta^2\bar\theta^2\Big\langle (\gamma^\mu)_{ab}\frac{i\partial_\mu}{2\partial^2} D^a e^{2V}\bar D^b \frac{1}{D^2 e^{2V}\bar D^2} - (\gamma^\mu)_{ab}\frac{i\partial_\mu}{2\partial^2} D^a e^{2V} D^2 \frac{1}{D^2 e^{2V}\bar D^2}\times\\
\times D^2 e^{2V}\bar D^b \frac{1}{D^2 e^{2V}\bar D^2} + \frac{1}{D^2 e^{2V}\bar D^2} D^a e^{2V}\frac{1}{D^2 e^{2V}\bar D^2} D_a e^{2V}\bar D^2 -\\
-\frac{1}{D^2 e^{2V}\bar D^2} D^a e^{2V}\bar D^2 \frac{1}{D^2 e^{2V}\bar D^2} D^2 e^{2V}\frac{1}{D^2 e^{2V}\bar D^2} D_a e^{2V}\bar D^2 +\\
+2\frac{1}{D^2 e^{2V}\bar D^2} D^a e^{2V}\bar D^b \frac{1}{D^2 e^{2V}\bar D^2} D^2 e^{2V}\bar D_b \frac{1}{D^2 e^{2V}\bar D^2} D_a e^{2V}\bar D^2 - \qquad (56)\\
-2\frac{1}{D^2 e^{2V}\bar D^2} D^a e^{2V}\bar D^2 \frac{1}{D^2 e^{2V}\bar D^2} D^2 e^{2V}\bar D^b \frac{1}{D^2 e^{2V}\bar D^2} D^2 e^{2V}\bar D_b \frac{1}{D^2 e^{2V}\bar D^2} D^b e^{2V}\bar D^2\Big\rangle.
\end{aligned}
$$

Here we take into account that all terms, in which an overall degree of θ and $\bar\theta$ is not 4, disappear after integrating over the anticommuting variables. In particular, due to this reason, the expression $\theta^2\bar\theta^2$ in the last identity can be moved to an arbitrary place of the trace. The sum of 2 first terms in Eq. (56) is 0, because it is proportional to

$$
\begin{aligned}
\mathrm{Tr}\,\frac{d}{d\ln\Lambda}\,\theta^2\bar\theta^2\Big[\bar\theta^b,\ \Big\langle (\gamma^\mu)_{ab}\frac{i\partial_\mu}{4\partial^2} D^a e^{2V}\bar D^2\frac{1}{D^2 e^{2V}\bar D^2}\Big\rangle\Big] + \mathrm{Tr}\,\frac{d}{d\ln\Lambda}\,\theta^2\bar\theta^2\Big\langle(\gamma^\mu)_{ab}\frac{i\partial_\mu}{4\partial^2}\times\\
\times D^a e^{2V}\bar D^2\frac{1}{D^2 e^{2V}\bar D^2}\frac{D^2\bar D^b}{16\partial^2}\Big\rangle = -\mathrm{Tr}\,\frac{d}{d\ln\Lambda}\,\theta^2\bar\theta^2\frac{D^2\bar D^2}{16^2\partial^2} = 0. \qquad (57)
\end{aligned}
$$

Using this equality, in brief notation (41) the expression for T_2 can be rewritten in the more compact form:

$$
\begin{aligned}
T_2 = \frac{1}{8}\frac{d}{d\ln\Lambda}\mathrm{Tr}\,\theta^2\bar\theta^2\Big\langle 2*(I_3)^a*(I_1)_a + 2*(\bar I_2)*(I_1)^a*(I_1)_a - *(I_2)^{ab}*(\bar I_1)_b*(I_1)_a -\\
- *(I_1)^a*(\bar I_1)^b*(\bar I_1)_b*(I_1)_a\Big\rangle. \qquad (58)
\end{aligned}
$$

To simplify it, we consider the trace of the commutator

$$
\begin{aligned}
0 = (\gamma^\mu)_{ab}\mathrm{Tr}\frac{d}{d\ln\Lambda}\Big[y^\mu, \theta^2\bar\theta^2\Big\langle\frac{1}{D^2 e^{2V}\bar D^2} D^a e^{2V}\bar D^b\Big\rangle\Big] =\\
= (\gamma^\mu)_{ab}\mathrm{Tr}\frac{d}{d\ln\Lambda}\Big[y^\mu, \theta^2\bar\theta^2\Big\langle\frac{D^2\bar D^2}{16\partial^2}\frac{1}{D^2 e^{2V}\bar D^2}\frac{D^2\bar D^2}{16\partial^2} D^a e^{2V}\bar D^b\Big\rangle\Big], \qquad (59)
\end{aligned}
$$

where $y^\mu \equiv x^\mu + i\bar\theta\gamma^\mu\gamma_5\theta/2$. (In the momentum representation this expression is evidently an integral of a total derivative with respect to the loop momentum.) For calculating this expression we will first use the identities

$$
[y^\mu, \bar D_a] = 0; \qquad [y^\mu, D_a] = 2i(\gamma^\mu\bar\theta)_a. \qquad (60)
$$

We obtain

$$
\begin{aligned}
0 = (\gamma^\mu)_{ab}\mathrm{Tr}\frac{d}{d\ln\Lambda}\,\theta^2\bar\theta^2\Big\langle\frac{2i}{D^2 e^{2V}\bar D^2}(\gamma^\mu\bar\theta)^a e^{2V}\bar D^b - \frac{4i}{D^2 e^{2V}\bar D^2}(\gamma^\mu\bar\theta)^c D_c e^{2V}\bar D^2\frac{1}{D^2 e^{2V}\bar D^2}\times\\
\times D^a e^{2V}\bar D^b + (\gamma^\mu)_{ab}\frac{2\partial_\mu}{\partial^2}\frac{1}{D^2 e^{2V}\bar D^2} D^a e^{2V}\bar D^b\Big\rangle = \mathrm{Tr}\frac{d}{d\ln\Lambda}\,\theta^2\bar\theta^2\Big\langle -\frac{8i}{D^2 e^{2V}\bar D^2}\bar\theta_b e^{2V}\bar D^b +\\
+\frac{8i}{D^2 e^{2V}\bar D^2}\bar\theta_b D_a e^{2V}\bar D^2\frac{1}{D^2 e^{2V}\bar D^2} D^a e^{2V}\bar D^b + (\gamma^\mu)_{ab}\frac{2\partial_\mu}{\partial^2}\frac{1}{D^2 e^{2V}\bar D^2} D^a e^{2V}\bar D^b\Big\rangle. \qquad (61)
\end{aligned}
$$

Then, we put together all θ and $\bar\theta$, commuting them with the covariant derivatives:

$$0 = \mathrm{Tr}\frac{d}{d\ln\Lambda}\,\theta^2\bar\theta^2\Big\langle\frac{1}{D^2e^{2V}\bar D^2}D^2e^{2V}\bar D^a\frac{1}{D^2e^{2V}\bar D^2}e^{2V}\bar D_a + \\ +\frac{1}{D^2e^{2V}\bar D^2}D^2e^{2V}\bar D_b\frac{1}{D^2e^{2V}\bar D^2}D_a e^{2V}\bar D^2\frac{1}{D^2e^{2V}\bar D^2}D^a e^{2V}\bar D^b\Big\rangle. \qquad (62)$$

In the brief notation this equality can be rewritten as

$$\mathrm{Tr}\,\frac{d}{d\ln\Lambda}\,\theta^2\bar\theta^2\Big\langle 4*(\bar I_3)^a*(\bar I_1)_a - *(I_2)^{ab}*(\bar I_1)_b*(I_1)_a\Big\rangle = 0. \qquad (63)$$

Taking into account that, evidently,

$$\mathrm{Tr}\,\frac{d}{d\ln\Lambda}\,\theta^2\bar\theta^2\Big\langle *(\bar I_3)^a*(\bar I_1)_a\Big\rangle = \mathrm{Tr}\,\frac{d}{d\ln\Lambda}\,\theta^2\bar\theta^2\Big\langle *(I_3)^a*(I_1)_a\Big\rangle, \qquad (64)$$

the result for the considered correlator can be simplified:

$$T_2 = \frac{1}{8}\mathrm{Tr}\frac{d}{d\ln\Lambda}\,\theta^2\bar\theta^2\Big\langle 2*(\bar I_2)*(I_1)^a*(I_1)_a - \frac{1}{2}*(I_2)^{ab}*(\bar I_1)_b*(I_1)_a - \\ - *(I_1)^a*(\bar I_1)^b*(\bar I_1)_b*(I_1)_a\Big\rangle. \qquad (65)$$

Unfortunately, we could not so far prove that this expression was 0. This is a key point in the proof of the new identity and, therefore, of the exact NSVZ β-function. Possibly, this expression is not an integral of total derivatives, but due to some reasons does not contribute to the three- and four-loop diagrams, which were calculated earlier. Then, it is necessary to modify the exact β-function by the following way:

$$\beta(\alpha) = \frac{\alpha^2}{\pi}\Big(1-\gamma(\alpha)-\delta(\alpha)\Big), \qquad (66)$$

where the function $\delta(\alpha)$ is defined as follows: If we define the operator

$$\hat O^a \equiv Z\frac{D^a\bar D^2}{2\partial^2}\phi^* e^{2V}\phi \qquad (67)$$

and write its correlator in the form

$$\langle \hat O^a_x\,\hat O^b_y\rangle = -\frac{i}{2\pi^2}C^{ab}\Delta(\alpha,\partial^2/\mu^2)\bar D^2\delta^8_{xy}, \qquad (68)$$

then

$$\delta\Big(d(\alpha,\mu/p)\Big) \equiv \frac{\partial}{\partial\ln p}\Delta(\alpha,\mu/p), \qquad (69)$$

where the function d is defined by Eq. (18).

5 Conclusion

In this paper we investigate an exact expression for the contribution of matter superfields to the Gell-Mann–Low function for the massless $N = 1$ supersymmetric electrodynamics. This investigation is based on substituting solutions of the Slavnov–Taylor identities into the Schwinger–Dyson equations. In particular, for contributions of matter superfields we try to prove an interesting feature, which was first noted in Ref. [3]: in supersymmetric theories the Gell-Mann–Low function is given by integrals of total derivatives. This is a fact that is responsible for the new identity, which was first proposed in Ref. [10]. This identity appears, if we require that the exact Gell-Mann–Low function coincides with the exact NSVZ β-function, and does not follow from any known symmetry of the theory. In this paper we tried to strictly derive the new identity for the massless case in the Abelian theory. The key point of the proof is a functional formulation of the new identity, proposed in Ref. [16]. Actually, the calculations, presented here, repeat qualitative speculations of Ref. [13], but there are essential differences in some points. Using the proposed method, it is possible to prove that some contributions to the new identity are really integrals of total derivatives and equal to 0. However, there are some contributions, which we could not not factorize to total derivatives. In principle, there are 2 possibilities: either the proof should be made differently and the Gell-Mann–Low function coincides with the exact NSVZ β-function, or the situation is similar to

$$\frac{987654321}{123456789} \approx 8.000000073, \tag{70}$$

i.e. the factorization into total derivatives is an accidental fact, appearing only in the lowest loops, and real structure of the result becomes clear only in the four-loop or higher approximation. Then, it is necessary to modify the expression for the β-function and add a contribution from the correlator of some composite operator.

If the new identity is true, then it is important to find a symmetry, responsible for the new identity. Existence of new symmetries was earlier proposed investigating finite $N = 1$ supersymmetric theories [17]. Possibly, these symmetries are somehow related with the AdS/CFT-correspondence [18], but so far they are not yet constructed. Nevertheless, the formulation of the new identity in terms of correlators for some composite operators suggests that these operators corresponds to fields in another theory. Then the equality of their correlator to 0 can follow from some symmetry of this theory.

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A Appendix: Why the contribution of the first term in Eq. (33) is 0

.

It is easy to see that the contribution of the first term in Eq. (33) to the new identity is 0. Really, let us consider, for example,

$$\frac{d}{d\ln\Lambda}\int d^8x\,d^8y\,\Big\langle D_x^2\mathbf{V}_x\frac{\bar D_x^2}{\partial^2}\phi_x^* e^{2V_x}\phi_x\,\frac{D_y^2\bar D_y^2}{\partial^2}(\phi_y^*\mathbf{V}_y)e^{2V_y}\phi_y\Big\rangle =$$
$$= \frac{d}{d\ln\Lambda}\int d^8x\,d^8y\,D_x^2\mathbf{V}_x\mathbf{V}_y\Big\langle\frac{\bar D_x^2}{\partial^2}\phi_x^* e^{2V_x}\phi_x\,\phi_y^*\frac{\bar D_y^2 D_y^2}{\partial^2}(e^{2V_y}\phi_y)\Big\rangle =$$
$$= 16i\frac{d}{d\ln\Lambda}\int d^8x\,d^8y\,D_x^2\mathbf{V}_x\mathbf{V}_y\frac{\bar D_x^2}{\partial^2}\frac{\delta}{\delta j_x^*}\frac{\delta}{\delta\phi_{0z}^*}\frac{\delta}{\delta j_y^*}\frac{\bar D_y^2 D_y^2}{\partial^2}\frac{\delta W}{\delta\phi_{0y}^*}\Big|_{z=x}, \qquad (71)$$

because, as earlier, all terms with two-point correlators are 0. (All two-point correlators are expressed in terms of the only function G, and this function is cancelled in their products. Hence, they disappear after differentiating with respect to $\ln\Lambda$.)

Now, let us prove that the last four-point correlator is 0. For this purpose we first use Eq. (26), and, then, express the derivative with respect to the additional source ϕ_0^* in terms of the derivative with respect to the field ϕ, using Schwinger–Dyson equation for the matter superfield (24). We obtain

$$\int d^8x\,d^8y\,D_x^2\mathbf{V}_x\mathbf{V}_y\frac{D_x^2}{\partial^2}\frac{\delta}{\delta j_x^*}\frac{\delta}{\delta\phi_{0z}^*}\frac{\delta}{\delta j_y^*}\frac{\bar D_y^2 D_y^2}{\partial^2}\frac{\delta W}{\delta\phi_{0y}^*}\Big|_{z=x} = -\frac{1}{2}\int d^8x\,d^8y\,D_x^2\mathbf{V}_x\mathbf{V}_y\,\times$$
$$\times\frac{D_x^2}{\partial^2}\frac{\delta}{\delta j_x^*}\frac{\delta}{\delta\phi_{0z}^*}\frac{\delta}{\delta j_y^*}\frac{\bar D_y^2}{\partial^2}\frac{\delta\Gamma}{\delta\phi_y^*}\Big|_{z=x} = \int d^8x\,d^8y\,D_x^2\mathbf{V}_x\mathbf{V}_y\frac{D_x^2}{\partial^2}\frac{\delta}{\delta j_x^*}\frac{\delta}{\delta\phi_{0z}^*}\frac{1}{\partial^2}\delta^4_{yw}\Big|_{z=x,w=y} = 0, \qquad (72)$$

because

$$\frac{\delta\Gamma}{\delta\phi^*} = -j^* \qquad (73)$$

even if fields and sources are not 0.

Completely similarly we obtain

$$\int d^8x\,d^8y\,\Big\langle D_x^b\mathbf{V}_x\frac{D_{xb}D_x^2}{\partial^2}\phi_x^* e^{2V_x}\phi_x\,\frac{D_y^2\bar D_y^2}{\partial^2}(\phi_y^*\mathbf{V}_y)e^{2V_y}\phi_y\Big\rangle = 0. \qquad (74)$$

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